

Static Deformation of Orthotropic Half-Space with Rigid Boundary due to Various Seismic Sources

Yogita Godara¹, Ravinder Kumar² and Mahabir Singh³

¹Research Scholar in Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonapat-131039, Haryana

²Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonapat-131039, Haryana

³Dhankhar, COE, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonapat-131039, Haryana (India)

E-mail: anjali18feb2010@gmail.com, 2sehrawatravinder@yahoo.co.in, 3msdhankhar@gmail.com.

Abstract—Closed form analytical expressions for displacements and stresses at any point of a homogeneous, orthotropic, perfectly elastic half-space with rigid boundary caused by two-dimensional seismic sources are obtained. The method consists of first finding the integral expressions for a homogeneous, orthotropic, perfectly elastic half-space from the corresponding expressions for an unbounded medium by applying suitable boundary conditions at the interface and then evaluating the integrals analytically.

Keywords: Rigid boundary, Static deformation, Orthotropic half-space.

1. INTRODUCTION

Generally, earthquakes are along geological faults which are surfaces of material discontinuity in the earth. To study the effect of faulting at a material discontinuity, many investigators considered the two half-space model. Steketee (1958a, b) [1-2] applied the elasticity theory of dislocations. Steketee dealt with a semi-infinite, non-gravitating, isotropic and homogenous medium. Homogeneity means that the medium is uniform throughout, whereas isotropy specifies that the elastic properties of the medium are independent of direction. Maruyama (1966) [6] calculated all sets of Green's function for obtaining displacements and stresses around faults in a half space. Freund and Barnett (1971) [3] obtained two dimensional surface deformation due to dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable. Singh and Garg (1986) [11] obtained the integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional seismic sources. Singh et. al (1991) [12] followed a similar procedure to obtain closed form analytical expression for the displacements and stresses at any point of either of two homogenous, isotropic, perfectly elastic half-spaces in welded contact due to two-dimensional sources.

Using the concept of orthotropic media, Singh (1986) [10], Garg and Singh (1987) [4], Pan (1989a) [7] studied the static deformation of a transversely isotropic multilayered half-space by surface loads. The problem of the static deformation of a transversely isotropic multilayered half-space by buried sources has been discussed by Pan (1989b) [8]. Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads has been investigated by Garg et al. (1991) [5]. Rani et al. (1991) [9] obtained the displacements and stresses at any point of a uniform half-space due to two-dimensional buried sources. Rani et al (2009) [13] obtained the closed-form expressions for the elastic residual field caused by a long dip-slip fault of finite width located in an isotropic half-space any point isotropic half-space in welded contact with orthotropic half-space. Godara et al. (2014) [16] derived the results for stresses and displacements due to two-dimensional seismic sources embedded in an isotropic half-space in smooth contact with an orthotropic half-space. Godara et al. (2014) [19] give the results for static deformation due to a long tensile fault of finite width in an isotropic half-space welded with an orthotropic half-space.

Singh et al. (2011) [14] obtained analytical expressions for stresses at an arbitrary point of homogenous, isotropic perfectly elastic half-space with rigid boundary caused by a long tensile fault of finite width. Malik et al. (2012,13), [15, 18] obtained the closed-form expressions for displacement and stress field for a uniform half-space with rigid boundary. Sahrawat et al. (2014) [19], obtained analytical expressions for stresses and displacements at an arbitrary point of homogenous, isotropic perfectly elastic half-space with rigid boundary caused by a long dip-slip fault of finite width. There is no literature for deformation of orthotropic half-space with rigid boundary and also our Earth's upper part is made up of rigid materials, so we consider a model that consists of a

dislocation in a homogeneous, orthotropic, perfectly elastic half-space in contact with a rigid half-space. This model is useful when the medium on the other side of the material discontinuity is very hard. We study the static deformation of a homogeneous, orthotropic, perfectly elastic half-space with rigid boundary caused by two-dimensional seismic sources.

2. THEORY

Let the Cartesian co-ordinates be denoted by $(x, y, z) \equiv (x_1, x_2, x_3)$ with z -axis vertically upwards. Consider an orthotropic elastic medium, with co-ordinate planes coinciding with the axis of symmetry and one plane of symmetry being horizontal, the stress-strain relation in matrix form is

$$\begin{bmatrix} p_{11} \\ p_{22} \\ p_{33} \\ p_{23} \\ p_{31} \\ p_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2e_{23} \\ 2e_{31} \\ 2e_{12} \end{bmatrix} \quad (1)$$

where, p_{ij} are the components of stress-tensor, e_{ij} are the components of strain-tensor, and c_{ij} are elastic constants of the medium.

A transversely isotropic elastic medium, with z -axis coinciding with the axis of symmetry, is a particular case of an orthotropic elastic medium for which $c_{22} = c_{11}$, $c_{23} = c_{13}$, $c_{55} = c_{44}$, $c_{66} = \frac{1}{2}(c_{11} - c_{12})$

and the number of independent elastic constants reduces from nine to five. When the medium is isotropic

$$c_{11} = c_{22} = c_{33} = \lambda + 2\mu,$$

$$c_{12} = c_{13} = c_{23} = \lambda, c_{44} = c_{55} = c_{66} = \mu$$

where, λ and μ are the Lamé's constants.

We consider a two dimensional approximation in which displacement component u_1, u_2, u_3 are independent of x so that $\partial/\partial x \equiv 0$. Under this assumption the plane strain problem ($u_1 = 0$) and anti-strain problem ($u_2 = 0$ and $u_3 = 0$) are decoupled and therefore, can be solved separately. The plane strain problem for an orthotropic medium can be solved in terms of the Airy stress function U such that Garg et al (1991)

$$p_{22} = \frac{\partial^2 U}{\partial z^2}, p_{33} = \frac{\partial^2 U}{\partial y^2},$$

$$p_{23} = -\frac{\partial^2 U}{\partial y \partial z} \quad (2)$$

$$\left(a^2 \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \left(b^2 \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) U = 0 \quad (3)$$

$$a^2 + b^2 = \frac{(c_{22}c_{33} - c_{23}^2 - 2c_{23}c_{44})}{c_{33}c_{44}}, a^2b^2 = \frac{c_{22}}{c_{33}} \quad (4)$$

Let there be a line source parallel to the x -axis passing through the point $(0, 0, h)$. As shown by Singh and Garg (1986) [11], the Airy stress function U_0 for a line source parallel to the x -axis passing through the point $(0, 0, h)$ in an unbounded orthotropic medium can be expressed in the form.

$$U_0 = \int_0^\infty [(L_0 e^{ak|z-h|} + M_0 e^{bk|z-h|}) \sin ky + P_0 e^{akz-h} + Q_0 e^{bkz-h} \cos ky] k^{-1} dk, \quad (5)$$

where, L_0, M_0, P_0, Q_0 are the source co-efficients for the source lying in the orthotropic half-space.

For a line source parallel to the x -axis acting at the point $(0, 0, h)$ of medium ($z \geq 0$) the Airy stress function in orthotropic half-space is a solution of Eq.(3) and may be taken to the form (assuming $a \neq b$)

$$U = U_0 + \int_0^\infty [(Le^{akz} + Me^{bkz}) \sin ky + (Pe^{akz} + Qe^{bkz}) \cos ky] k^{-1} dk, \quad (6)$$

The constants L, M, P, Q etc. are to be determined from the boundary conditions.

The displacements for the orthotropic medium are given by Garg et al., (1991) [5]

$$u_2 = \frac{1}{\Delta} \int (c_{33}p_{22} - c_{23}p_{33}) dy, \\ u_3 = \frac{1}{\Delta} \int (c_{22}p_{33} - c_{23}p_{22}) dz, \quad (7)$$

where,

$$\Delta = c_{22}c_{33} - c_{23}^2 \quad (8)$$

From equations (2), (6), (7), the stresses and the displacements are found to be

$$p_{22} = \int_0^\infty [(a^2 L_0 e^{ak|z-h|} + b^2 M_0 e^{bk|z-h|} + a^2 L e^{akz} + b^2 M e^{bkz}) \sin ky + (a^2 P_0 e^{ak|z-h|} + b^2 Q_0 e^{bk|z-h|} + a^2 P e^{akz} + b^2 Q e^{bkz}) \cos ky] k dk \quad (9)$$

$$p_{23} = \int_0^\infty [(\mp(aL_0e^{ak|z-h|} + bM_0e^{bk|z-h|}) + aLe^{akz} + bMe^{bkz}) \cos ky + (\pm(aP_0e^{ak|z-h|} + bQ_0e^{bk|z-h|}) + aPe^{akz} + bQe^{bkz}) \sin ky] k dk \quad (10)$$

$$p_{33} = - \int_0^\infty [(L_0e^{ak|z-h|} + M_0e^{bk|z-h|} + Le^{akz} + Me^{bkz}) \sin ky + (P_0e^{ak|z-h|} + Q_0e^{bk|z-h|} + Pe^{akz} + Qe^{bkz}) \cos ky] k dk \quad (11)$$

$$u_2 = \int_0^\infty [-(r_1L_0e^{ak|z-h|} + r_2M_0e^{bk|z-h|} + r_1Le^{akz} + r_2Me^{bkz}) \cos ky + (r_1P_0e^{ak|z-h|} + r_2Q_0e^{bk|z-h|} + r_1Pe^{akz} + r_2Qe^{bkz}) \sin ky] dk \quad (12)$$

$$u_3 = - \int_0^\infty [(\pm(s_1L_0e^{ak|z-h|} + s_2M_0e^{bk|z-h|}) + s_1Le^{akz} + s_2Me^{bkz}) \sin ky + (\pm(s_1P_0e^{ak|z-h|} + s_2Q_0e^{bk|z-h|}) + s_1Pe^{akz} + s_2Qe^{bkz}) \cos ky] dk \quad (13)$$

Where,

$$r_1 = \frac{c_{33}a^2 + c_{23}}{\Delta}, r_2 = \frac{c_{33}b^2 + c_{23}}{\Delta},$$

$$s_1 = \frac{c_{23}a + c_{22}}{a^2\Delta}, s_2 = \frac{c_{23}b + c_{22}}{b^2\Delta},$$

$$\Delta = (c_{22}c_{33} - c_{23}^2), \quad (14)$$

It is noticed from appendix for source coefficients that the coefficients L_0, M_0, P_0, Q_0 might have different values for $z > h$ and $z < h$; let L^-, M^-, P^- , and Q^- be the values of L_0, M_0, P_0 , and Q_0 respectively, valid for $z < h$.

We assume that the surface of the half-space $z \geq 0$ is with rigid boundary. Therefore, the boundary conditions are

$$u_2 = 0 \text{ and } u_3 = 0 \text{ at } z = 0 \quad (15)$$

Using equations (12), (13) and (15), we will get the following system of equations:

$$\begin{aligned} r_1L^-e^{akh} + r_2M^-e^{bkh} + r_1L + r_2M &= 0, \\ -s_1L^-e^{akh} - s_2M^-e^{bkh} + s_1L + s_2M &= 0, \\ r_1P^-e^{akh} + r_2Q^-e^{bkh} + r_1P + r_2Q &= 0, \\ -s_1P^-e^{akh} - s_2Q^-e^{bkh} + s_1P + s_2Q &= 0, \end{aligned} \quad (16)$$

Solving the system for L, M, P and Q, we get

$$\begin{aligned} L &= t_1L^-e^{akh} + t_3M^-e^{bkh}, \\ M &= -t_2L^-e^{akh} - t_1M^-e^{bkh}, \\ P &= t_1P^-e^{akh} + t_3Q^-e^{bkh}, \\ Q &= -t_2P^-e^{akh} - t_1Q^-e^{bkh} \end{aligned} \quad (17)$$

where,

$$\begin{aligned} t_1 &= \frac{r_1s_2 + s_1r_2}{r_2s_1 - r_1s_2}, t_2 = \frac{2r_1s_1}{r_2s_1 - r_1s_2}, \\ t_3 &= \frac{2r_2s_2}{r_2s_1 - r_1s_2} \end{aligned} \quad (18)$$

Putting the values of the constants L_1, M_1, P_1 , etc. in Eqs. (10) and (11), we get the integral expressions for the Airy stress function in the two media. These integrals can be evaluated analytically using the standard integrals given in appendix. The displacements and stresses can be obtained similarly. Using the notation ($z \neq h$).

$$R_1^2 = y^2 + a^2(z-h)^2, R_2^2 = y^2 + b^2(z-h)^2,$$

$$S_1^2 = y^2 + a^2(z+h)^2, S_2^2 = y^2 + a^2(z+h)^2,$$

$$T_1^2 = y^2 + (az+bh)^2, T_2^2 = y^2 + (bz+ah)^2, \quad (19)$$

The final results are given below.

$$\begin{aligned} U &= L_0 \tan^{-1} \left(-\frac{y}{a|z-h|} \right) + M_0 \tan^{-1} \left(-\frac{y}{b|z-h|} \right) \\ &\quad - P_0 \ln R_1 - Q_0 \ln R_2 \\ &\quad + L^- \left[t_1 \tan^{-1} \left(-\frac{y}{a(z+h)} \right) \right. \\ &\quad \left. - t_2 \tan^{-1} \left(-\frac{y}{(bz+ah)} \right) \right] \\ &\quad + M^- \left[-t_1 \tan^{-1} \left(-\frac{y}{b(z+h)} \right) + t_3 \tan^{-1} \left(-\frac{y}{(az+bh)} \right) \right] \\ &\quad - P^- [t_1 \ln S_1 - t_2 \ln T_2] \\ &\quad + Q^- [t_1 \ln S_2 - t_3 \ln T_1] \end{aligned} \quad (20)$$

$$\begin{aligned} p_{22} &= -2a^3L_0 \frac{y|z-h|}{R_1^4} - 2b^3M_0 \frac{y|z-h|}{R_2^4} \\ &\quad + P_0 \frac{a^2}{R_1^2} \left[\frac{2a^2(z-h)^2}{R_1^2} - 1 \right] + Q_0 \frac{b^2}{R_2^2} \left[\frac{2b^2(z-h)^2}{R_2^2} - 1 \right] \\ &\quad - 2L^- \left[a^3t_1 \frac{y(z+h)}{S_1^4} - b^2t_2 \frac{y(bz+ah)}{T_2^4} \right] \\ &\quad - 2M^- \left[a^2t_3 \frac{y(az+bh)}{T_1^4} - b^3t_1 \frac{y(z+h)}{S_2^4} \right] \\ &\quad + P^- \left[\frac{a^2t_1}{S_1^2} \left(\frac{2a^2(z+h)^2}{S_1^2} - 1 \right) - \frac{b^2t_2}{T_2^2} \left(\frac{2(bz+ah)^2}{T_2^2} - 1 \right) \right] \\ &\quad + Q^- \left[\frac{a^2t_3}{T_1^2} \left(\frac{2(az+bh)^2}{T_1^2} - 1 \right) \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{b^2 t_1}{S_2^2} \left(\frac{2b^2(z+h)^2}{S_2^2} - 1 \right) \Bigg] \quad (21) \\
p_{23} = & \mp L_0 \frac{a}{R_1^2} \left[\frac{2a^2(z-h)^2}{R_1^2} \right. \\
& - 1 \Bigg] \mp M_0 \frac{b}{R_2^2} \left[\frac{2b^2(z-h)^2}{R_2^2} - 1 \right] \\
& \mp 2a^2 P_0 \frac{y|z-h|}{R_1^4} \\
& \mp 2b^2 Q_0 \frac{y|z-h|}{R_2^4} \\
& + L^- \left[\frac{at_1}{S_1^2} \left(\frac{2a^2(z+h)^2}{S_1^2} - 1 \right) \right. \\
& - \frac{bt_2}{T_2^2} \left(\frac{2(bz+ah)^2}{T_2^2} - 1 \right) \Bigg] \\
& + M^- \left[\frac{at_3}{T_1^2} \left(\frac{2(az+bh)^2}{T_1^2} - 1 \right) \right. \\
& - \frac{bt_1}{S_2^2} \left(\frac{2b^2(z+h)^2}{S_2^2} - 1 \right) \Bigg] \\
& - 2P^- \left[\frac{a^2 t_1}{S_1^4} \frac{y(z+h)}{S_1^4} \right. \\
& - \left. \frac{bt_2}{T_2^4} \frac{y(bz+ah)}{T_2^4} \right] \\
& - 2Q^- \left[\frac{at_3}{T_1^4} \frac{y(az+bh)}{T_1^4} \right. \\
& - \left. \frac{b^2 t_1}{S_2^4} \frac{y(z+h)}{S_2^4} \right] \quad (22)
\end{aligned}$$

$$\begin{aligned}
p_{33} = & 2aL_0 \frac{y|z-h|}{R_1^4} + 2bM_0 \frac{y|z-h|}{R_2^4} \\
& - P_0 \frac{1}{R_1^2} \left[\frac{2a^2(z-h)^2}{R_1^2} \right. \\
& - 1 \Bigg] - Q_0 \frac{1}{R_2^2} \left[\frac{2b^2(z-h)^2}{R_2^2} - 1 \right] \\
& + 2L^- \left[\frac{at_1}{S_1^4} \frac{y(z+h)}{S_1^4} \right. \\
& - \left. \frac{t_2}{T_2^4} \frac{y(bz+ah)}{T_2^4} \right] \\
& - 2M^- \left[\frac{t_3}{T_1^4} \frac{y(az+bh)}{T_1^4} \right. \\
& - \left. \frac{bt_1}{S_2^4} \frac{y(z+h)}{S_2^4} \right] \\
& - P^- \left[\frac{t_1}{S_1^2} \left(\frac{2a^2(z+h)^2}{S_1^2} - 1 \right) \right. \\
& - \left. \frac{t_2}{T_2^2} \left(\frac{2(bz+ah)^2}{T_2^2} - 1 \right) \right]
\end{aligned}$$

$$\begin{aligned}
& - Q^- \left[\frac{t_3}{T_1^2} \left(\frac{2(az+bh)^2}{T_1^2} - 1 \right) \right. \\
& - \left. \frac{t_1}{S_2^2} \left(\frac{2b^2(z+h)^2}{S_2^2} - 1 \right) \right] \quad (23)
\end{aligned}$$

$$\begin{aligned}
u_2 = & r_1 L_0 \frac{a|z-h|}{R_1^2} + r_2 M_0 \frac{b|z-h|}{R_2^2} + r_1 P_0 \frac{y}{R_1^2} \\
& + r_2 Q_0 \frac{y}{R_2^2} \\
& + L^- \left[r_1 t_1 \frac{a(z+h)}{S_1^2} \right. \\
& - \left. r_2 t_2 \frac{(bz+ah)}{T_2^2} \right] \\
& + M^- \left[r_1 t_3 \frac{(az+bh)}{T_1^2} \right. \\
& - \left. r_2 t_1 \frac{b(z+h)}{S_2^2} \right] \\
& + P^- \left[r_1 t_1 \frac{y}{S_1^2} - r_2 t_2 \frac{y}{T_2^2} \right] \\
& + Q^- \left[r_1 t_3 \frac{y}{T_1^2} \right. \\
& - \left. r_2 t_1 \frac{y}{S_2^2} \right] \quad (24)
\end{aligned}$$

$$\begin{aligned}
u_3 = & \mp s_1 L_0 \frac{y}{R_1^2} \mp s_2 M_0 \frac{y}{R_2^2} \mp s_1 P_0 \frac{a|z-h|}{R_1^2} \\
& \mp s_2 Q_0 \frac{b|z-h|}{R_2^2} \\
& - L^- \left[s_1 t_1 \frac{y}{S_1^2} - s_2 t_2 \frac{y}{T_2^2} \right] \\
& - M^- \left[s_1 t_3 \frac{y}{T_1^2} - s_2 t_1 \frac{y}{S_2^2} \right] \\
& - P^- \left[s_1 t_1 \frac{a(z+h)}{S_1^2} \right. \\
& - \left. s_2 t_2 \frac{(bz+ah)}{T_2^2} \right] \\
& - Q^- \left[s_1 t_3 \frac{(az+bh)}{T_1^2} \right. \\
& - \left. s_2 t_1 \frac{b(z+h)}{S_2^2} \right] \quad (25)
\end{aligned}$$

We have derived the generalised expressions for stresses and displacements for an orthotropic half-space with rigid boundary due to any seismic- source lying in it. Knowing the seismic co-efficients (for the source lying in the orthotropic half-space) L_0, M_0, P_0, Q_0 we can find out the stresses and displacements for orthotropic half-space and can explore the results for various half-spaces numerically and graphically.

3. ACKNOWLEDGEMENT

I, Yogita Godara (JRF), am grateful to the Council of Scientific and Industrial Research, New Delhi for financial support.

REFERENCES

- [1] Steketee, J.A., On Volterra's Dislocations in A Semi-infinite Elastic Medium, *Can. J. Phys.*, Vol. 36, 192-205, 1958a.
- [2] Steketee, J.A, Some Geophysical Applications of the Elasticity Theory of Dislocations, *Can. J. Phys.*, vol. 36, 1168-1198, 1958b.
- [3] Freund, L. B. and D. M. Barnett, A two-dimensional analysis of surface deformation due to dip-slip faulting, *Bull. Seismol. Soc. Am.*, 66, 667-675, 1976.
- [4] Garg, N. R. and S. J. Singh, 2-D response of a transversely isotropic multilayered half-space to surface loads, *Indian J. Pure Appl. Maths.*, 18, 763-777, 1987.
- [5] Garg, N. R., S. J. Singh, and S. Manchanda, Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads, *Proc. Ind. Acad. Sci. (Earth Planet. Sci.)*, 100, 205-218, 1991.
- [6] Maruyama, T., On two-dimensional elastic dislocations in an infinite and semi-infinite medium, *Bull. Earthq. Res. Inst., Univ. Tokyo*, 44, 811-871, 1966.
- [7] Pan, E., Static response of a transversely isotropic and layered half-space to general surface loads, *Phys. Earth Planet. Inter.*, 54, 353-363, 1989a.
- [8] Pan, E., Static response of a transversely isotropic and layered half-space to general dislocation sources, *Phys. Earth Planet. Inter.*, 58, 103-117, 1989b.
- [9] Rani, S., S. J. Singh, and N. R. Garg, Displacements and stresses at any point of a uniform half-space due to two-dimensional buried sources, *Phys. Earth Planet. Inter.* 65, 276-286, 1991.
- [10] Singh, S. J., Static deformation of a transversely isotropic multilayered half-space by surface loads, *Phys. Earth Planet. Inter.*, 42, 263-273, 1986.
- [11] Singh, S. J. and N. R. Garg, On the representation of two-dimensional seismic sources, *Acta Geophys. Pol.*, 34, 1-12, 1986.
- [12] Singh, S. J., S. Rani, and N. R. Garg, Displacements and stresses in two welded half-spaces due to two-dimensional sources, *Phys. Earth. Planet. Inter.*, 1991.
- [13] Rani, S. et al., Static deformation due to dip-slip fault of finite width embedded in an isotropic half-space in welded contact with an orthotropic half space, *Sadhana* Vol. 34, Part 6, pp. 887-902, *Indian Academy of Sciences*, 2009.
- [14] Singh, J., Malik, M. and Singh, M., Deformation of a uniform Half-space with Rigid Boundary Due to Long Tensile Fault, *ISET Journal of Earthquake Technology*, Vol. 48, 2011.
- [15] Malik, M., Singh, M., Singh J., Static Deformation of a Uniform Half-space with Rigid Boundary due to dipping fault at 45°, *Journal of computing Technologies*, 26-34, 2012.
- [16] Godara, Y., Sahrawat, R.K., Singh, M., Static deformation due to two-dimensional seismic sources embedded in an isotropic half-space in smooth contact with an orthotropic half-space, *Global Journal of Mathematical Analysis*, 2 (3), 169-183, 2014.
- [17] Godara, Y., Sahrawat, R.K., Singh, M., Static deformation due to a long tensile fault of finite width in an isotropic half-space welded with an orthotropic half-space, *Bulletin of Mathematical Sciences & applications*, Vol.3 No. 3, 13-33, 2014.
- [18] Malik, M., Singh, M., Singh J., Static Deformation of a Uniform Half-space with Rigid Boundary due to a Vertical Dip-Slip Line Source, *IOSR JM*, Vol. 4, 26-37, 2013.
- [19] Sahrawat, R.K., Godara, Y., Singh, M.,: 'Static deformation of a uniform half- space with rigid boundary due to a long dip-slip fault of finite width', *Inter. J. Engg. Tech. Research*, Volume-2, Issue-5, 189-194, May 2014